

Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

October/November 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1 (a) Use the list of formulae (MF19) to find $\sum_{r=1}^n r(r+2)$ in terms of n , simplifying your answer. [2]

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- (b) Express $\frac{1}{r(r+2)}$ in partial fractions and hence find $\sum_{r=1}^n \frac{1}{r(r+2)}$ in terms of n . [4]

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- (c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+2)}$. [1]

2 The equation $x^4 + 3x^2 + 2x + 6 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find a quartic equation whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}, \frac{1}{\delta^2}$ and state the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}$.

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) Find the value of $\beta^2\gamma^2\delta^2 + \alpha^2\gamma^2\delta^2 + \alpha^2\beta^2\delta^2 + \alpha^2\beta^2\gamma^2$. [3]

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- (c) Find the value of $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} + \frac{1}{\delta^4}$. [2]

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- 3** The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$, where k is a constant and $k \neq 0$ or 1 .

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

(b) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

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(c) Show that the invariant points of the transformation represented by $\mathbf{M}$ lie on the line $y = \frac{k^2}{1-k}x$. [4]

[illegible]

- (d) The triangle ABC in the x - y plane is transformed by $\mathbf{M}$ onto triangle DEF .

Find the value of k for which the area of triangle DEF is equal to the area of triangle ABC . [2]

- 4 The function f is such that $f''(x) = f(x)$.

Prove by mathematical induction that, for every positive integer n ,

$$\frac{d^{2n-1}}{dx^{2n-1}}(xf(x)) = xf'(x) + (2n-1)f(x). \quad [7]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

5 The curve C has polar equation $r = a \sec^2 \theta$, where a is a positive constant and $0 \leq \theta \leq \frac{1}{4}\pi$.

- (a) Sketch C , stating the polar coordinates of the point of intersection of C with the initial line and also with the half-line $\theta = \frac{1}{4}\pi$. [3]

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- (b) Find the maximum distance of a point of C from the initial line. [2]

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- (c) Find the area of the region enclosed by C , the initial line and the half-line $\theta = \frac{1}{4}\pi$. [4]

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(d) Find, in the form $y = f(x)$, the Cartesian equation of C . [3]

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- 6** The lines l_1 and l_2 have equations $\mathbf{r} = 2\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ respectively.

The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2 .

- (a) Find the length PQ . [5]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced and extend across the entire width of the page, providing a guide for letter height and placement. There is no text or other markings on the page.

The plane Π_1 contains PQ and l_1 .

The plane Π_2 contains PQ and l_2 .

- (b) (i) Write down an equation of Π_1 , giving your answer in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$. [1]

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- (ii) Find an equation of Π_2 , giving your answer in the form $ax + by + cz = d$. [4]

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- (c) Find the acute angle between Π_1 and Π_2 . [5]

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7 The curve C has equation $y = \frac{x^2 - x}{x + 1}$.

(a) Find the equations of the asymptotes of C .

[3]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find the exact coordinates of the stationary points on C .

[4]

[illegible]

- (c) Sketch C , stating the coordinates of any intersections with the axes. [3]

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- (d) Sketch the curve with equation $y = \left| \frac{x^2 - x}{x + 1} \right|$ and find in exact form the set of values of x for which $\left| \frac{x^2 - x}{x + 1} \right| < 6$. [5]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

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